New collective mode due to collisional coupling

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Starting from a nonmarkovian conserving relaxation time approximation for collisions we derive coupled dispersion relations for asymmetric nuclear matter. The isovector and isoscalar modes are coupled due to asymmetric nuclear meanfield acting on neutrons and protons differently. A further coupling is observed by collisional correlations. The latter one leads to the appearance of a new soft mode besides isoscalar and isovector modes in the system. We suggest that this mode might be observable in asymmetric systems. This soft mode approaches the isovector mode for high temperatures. At the same time the isovector mode remains finite and approaches a constant value at higher temperatures showing a transition from zero sound like damping to first sound. The damping of the new soft mode is first sound like at all temperatures.

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The investigation of collective excitations in asymmetric nuclear matter is of current interest for experiments with nuclei far from β- stability, [1] and citations therein. We consider here a Fermi gas model consisting of a number of different species (neutrons, protons, etc) interacting with the own specie and with other ones. The interaction between different sorts of particles is important to consider if we want to include friction between different streams of isospin components. Especially the isospin current may not be conserved by this way. We neglect explicitly shell effects and concentrate only on bulk matter properties. Let us start with a set of coupled quantum kinetic equations for the reduced density operator ρ_a for the specie a

$$\partial_t \rho_a(t) = i[\rho_a, \mathcal{E}_a + \mathcal{U}(t)_a] - \sum_b \int_0^t dt' \frac{\rho_a(t') - \tilde{\rho}_b(t')}{\tau_{ab}(t-t')} \quad (1)$$

where $\mathcal{E} = \mathcal{P}^2/2m$ denotes the kinetic energy operator and $\mathcal{U}$ the mean field operator and the external field which is assumed to be a nonlinear function of the density. We have approximated the collision integral by a non-Markovian relaxation time [8]. The memory effects condensed in the frequency dependent relaxation time turned out to be necessary to reproduce damping of zero sound [2,3]. It accounts for the fact that during a two particle collision a collective mode can couple to the scattering process. Consequently, the dynamical relaxation time represents the physical content of a hidden three particle process and is equivalent to the memory effects.

We have further assumed the relaxation with respect to the local equilibrium $\tilde{\rho}_b$ of any specie in the system. The relaxation of the actual distribution of specie a is driven by the local equilibrium of all the other components. The cross coupling is realized by nondiagonal relaxation times τ_{ab}. We specify the local equilibrium by a small deviation of the chemical potential of specie a [4] compared with the global equilibrium

$$< k|\tilde{\rho}_a|k' >= f_a(k)\delta_{kk'} - \frac{f_a(k) - f_a(k')}{\epsilon_a(k) - \epsilon_a(k')}\delta\mu_a(k-k') \quad (2)$$

with $< k|\mathcal{E}|k' >= \epsilon(k)$. The equilibrium distributions $< k|\rho_a^0|k' >= f_a(k)\delta_{kk'}$ are the corresponding Fermi functions with chemical potential μ_a and the normalization to density $n_a = 2\int \frac{dp}{(2\pi)^3} f_a(p)$. The local equilibrium specified by $\delta\mu_a$ is determined if we impose the density balance to be fulfilled separately for each specie current J_a which reads in Wigner coordinates $k = p + q/2$ and $k' = p - q/2$

$$\omega \delta n_a(q,\omega) = q\delta J_a(q,\omega). \quad (3)$$

From this equation we derive the following matrix equation for $\delta\mu_a$

$$\Pi_a(q,0)\delta\mu_a = \sum_b \left\{\frac{1}{\tau}\right\}^{-1}_{ab} \left(B_b\delta(q) + \frac{\delta n_b}{\tau_b}\right) \quad (4)$$

where $B_a = \sum_b \frac{n_b - n_a}{\tau_{ab}}$,

$$\frac{1}{\tau_a} = \sum_b \frac{1}{\tau_{ab}}, \quad (5)$$

and the partial polarization function of specie a is

$$\Pi_a(q,\omega) = 2\int \frac{dp}{(2\pi)^3} \frac{f_a(p+\frac{q}{2}) - f_a(p-\frac{q}{2})}{\epsilon_a(p+\frac{q}{2}) - \epsilon_a(p-\frac{q}{2}) - \omega}. \quad (6)$$

The factor 2 in front of the integral accounts for the spin degeneracy. Eq. (4) generalizes eq. 6 of [4] to multicomponent systems.

Now we linearize the kinetic equation (1) around the total equilibrium ρ_a^0 which we express as

$$\delta\rho_a = \rho_a - \rho_a^0 = \rho_a - \tilde{\rho}_b + \tilde{\rho}_b - \rho_a^0. \tag{7}$$

This provides us with the density variation δn due to an external field $U^{\rm ext}$ which is connected with the polarization of the system via $\Pi\delta n = U^{\rm ext}$. The obtained polarization function has now a matrix structure. The poles of the polarization function represent the collective modes in the system. For a two component system, e.g. neutrons with density n_n and protons with density n_p, we have the density variation of the mean field

$$\delta U_a = \alpha_{an}\delta n_n + \alpha_{ap}\delta n_p \tag{8}$$

and we obtain for the collective modes the dispersion relation

$$\begin{aligned}&(1-\Pi_n^{\rm M}\alpha_{nn})(1-\Pi_p^{\rm M}\alpha_{pp})\\ &-(D_{np}+\Pi_n^{\rm M}\alpha_{np})(D_{pn}+\Pi_p^{\rm M}\alpha_{pn}) = 0.\end{aligned} \tag{9}$$

The generalization of the Mermin polarization function [4] to a multicomponent system is derived with the inclusion of nonmarkovian (frequency dependent) relaxation times as

$$\Pi_a^{\rm M} = \frac{\Pi_a(\omega+\frac{i}{\tau_a})}{1-\frac{i}{\omega\tau_a+i}(1-C_{aa})}. \tag{10}$$

An additional coupling in (9) occurs due to asymmetry and collisions

$$D_{np} = \frac{\tau_n}{\tau_p}\frac{C_{np}}{C_{nn}-i\omega\tau_n}. \tag{11}$$

The D_{pn} are given by interchanging sort indices. The matrix C_{ab} is expressed as

$$C_{ab} = \sum_c \left\{\frac{1}{\tau}\right\}_{ac} \frac{\Pi_c((\omega+\frac{i}{\tau_a})\frac{m_a}{m_c})}{\Pi_c(0)} \left\{\frac{1}{\tau}\right\}_{cb}^{-1}. \tag{12}$$

The term D_{np} is vanishing for symmetric matter, i.e. for equal densities of species as well as for vanishing collision integral $\tau\to\infty$. Therefore we call this term asymmetry coupling term further on.

The dispersion relation (9) is similar to the one derived recently in [1] if we neglect the collisional coupling D_{np}. The latter one has also been discussed in plasma two-stream instabilities [5]. Here we present a more general dispersion including correlational coupling. The polarization function includes collisions within a conserving approximation [6].

Before we apply this dispersion relation to nuclear matter let us consider a special case. We assume symmetric nuclear mean fields $\alpha_{nn} = \alpha_{pp} = \alpha_1$ and $\alpha_{np} = \alpha_{pn} = \alpha_2$ and neglecting the collisions we obtain

$$1-(\alpha_1\pm\alpha_2)\frac{\Pi(\omega+\frac{i}{\tau})}{1-\frac{i}{\omega\tau+i}(1-\frac{\Pi(\omega+\frac{i}{\tau})}{\Pi(0)})} = 0. \tag{13}$$

These are the decoupled dispersion relations for the isovector mode $\alpha_1-\alpha_2$ and the isoscalar mode $\alpha_1+\alpha_2$.

Summarizing, we see that there appear two kinds of coupling: (i) The coupling of modes between isovector and isoscalar ones due to different mean fields and (ii) an explicit correlational coupling of asymmetric nuclear matter due to collisional correlations, which is condensed in D_{np}. We have presented a general dispersion relation for the multicomponent system including known special cases.

In the following we will apply this expression for the damping of giant dipole resonances in asymmetric nuclear matter. We assume a wave vector for giant dipole resonances corresponding to the Steinwedel and Jensen [7] model $q = \frac{2.1}{R}$ with the nuclear radius $R = 1.13\,A^{1/3}$ fm connected to mass number A. This wave vector has very low values compared with the Fermi wave vector. Therefore it allows us to expand the Mermin polarization function with respect to small qv_c/ω ratios where v_c is the sound velocity. The frequency dependence of the dynamical (memory) relaxation times is derived using a Sommerfeld expansion [8]

$$\frac{1}{\tau_{ab}(\omega)} = \frac{1}{\tau_{ab}(0)}\left(1+\frac{3}{4}\left(\frac{\omega}{\pi T}\right)^2\right) \tag{14}$$

for a, b neutrons or protons respectively. The markovian relaxation time was given in terms of the cross section σ_{ab} between specie a and b as $\tau_{ab}^{-1} = \frac{4m}{3\hbar^3}\sigma_{ab}T^2$.

The dispersion relation (9) takes then the form of a polynomial of tenth (six) order corresponding to the inclusion of memory (in)dependent relaxation times via (14)

$$\begin{aligned}0 =& (\omega(\omega+\frac{i}{\tau_n})-c_{nn}^2q^2)(\omega(\omega+\frac{i}{\tau_p})-c_{pp}^2q^2)\\ &-\left(c_{np}^2+i\frac{\bar{c}_{np}^2}{(\omega+\frac{i}{\tau_n})\tau_p}\right)\left(c_{pn}^2+i\frac{\bar{c}_{pn}^2}{(\omega+\frac{i}{\tau_p})\tau_n}\right)q^4\end{aligned} \tag{15}$$

with the definition (5) and (14). Here the partial sound velocities are

$$\begin{aligned}c_{ab}^2 &= \alpha_{ab}\frac{n_a(\mu_a)}{m}\\ \bar{c}_{ab}^2 &= \frac{1}{m}\frac{E_n-E_p}{\frac{\tau_{np}}{\tau_{pp}}-\frac{\tau_{nn}}{\tau_{pn}}}\end{aligned} \tag{16}$$

and denoting the Fermi energy with ϵ_f^a

$$E_a = T\frac{f_{3/2}({\rm e}^{\beta\epsilon_f^a})}{f_{1/2}({\rm e}^{\beta\epsilon_f^a})}. \tag{17}$$

We observe that the new collisional coupling represented by $\bar{c}_{ab}^2$ vanishes if either the diagonal friction τ_{aa}^{-1} or the

nondiagonal friction τ_{ab}^{-1} vanishes or the system is symmetric $E_n = E_p$. This underlines that we have a new coupling due to collisional correlations and asymmetry.

We use as an illustrative example the following mean field parameterization of Vautherin [9,10]

$$U_a = t_0 \left[(1 + \frac{x_0}{2})(n_n + n_p) - (x_0 + \frac{1}{2}) n_a \right] + \frac{t_3}{4}((n_n + n_p)^2 - n_a^2) \tag{18}$$

with $n_a = n_n, n_p$ the density of neutrons or protons, respectively. The corresponding mean field deviations can easily be computed via $\alpha_{ab} = \partial U_a / \partial n_b$. The Coulomb interaction leads to an additional contribution for the proton meanfield

$$U_p^C(q) = \frac{4\pi e^2}{q^2} n_p(q). \tag{19}$$

The here used model parameters reproduce the Weizsäcker formula

$$\frac{E}{A} = -a_1 + \frac{a_2}{A^{1/3}} + \frac{a_3 Z^2}{A^{4/3}} + a_4 \delta^2 \tag{20}$$

by the volume energy $a_1 = 15.68$ MeV, Coulomb energy $a_3 = 0.717$ MeV and the symmetry energy $a_4 = 28.1$ MeV with the asymmetry parameter

$$\delta = \frac{n_n - n_p}{n_n + n_p}. \tag{21}$$

In figure 1 we have plotted the solution of the dispersion relation (15) for ^{11}Be versus excitation energy. The kinetic energy is linked to a temperature within the Fermi liquid model via Sommerfeld expansion

$$\frac{E}{A} = \frac{3}{5}\epsilon_f \left(\frac{(1+\delta)^{5/3} + (1-\delta)^{5/3}}{2} + \frac{5}{12}\pi^2 (\frac{T}{T_f})^2 \frac{(1+\delta)^{1/3} + (1-\delta)^{1/3}}{2} \right). \tag{22}$$

This connection between temperature and excitation energy is only valid for a continuous Fermi liquid model. For the small nuclei like ^{11}Be, the concept of temperature is questionable. Some improvement one can obtain by the definition of temperature via the logarithmic derivative of the density of states [11]

$$T^{-1} = \frac{1}{\rho}\frac{\partial \rho}{\partial E_{\rm ex}} = -\frac{5}{4} E_{\rm ex}^{-1} + \pi (\frac{A}{4\epsilon_f E_{\rm ex}})^{1/2} \tag{23}$$

which provides $E_{\rm ex} \approx \frac{1}{4}(E/A)$ in comparison with (22) for small temperatures. We use this temperature to demonstrate possible collective bulk features in an exploratory sense. Of course, the surface energy and shell effects cannot be neglected for realistic calculations.

In figure 1 we plot the occurring modes for a weak asymmetric case of ^{11}B. We observe that the isovector mode is decreasing with increasing excitation energy and vanishes at about 5 MeV. The corresponding isoscalar mode is already vanishing at 4 MeV. In general all energies appear as symmetric solutions with positive and negative energy where negative energy solutions are ruled out as unphysical. The corresponding damping is degenerated up to the point of vanishing energy. Above this temperature the damping of isoscalar and isovector modes become twofolded.

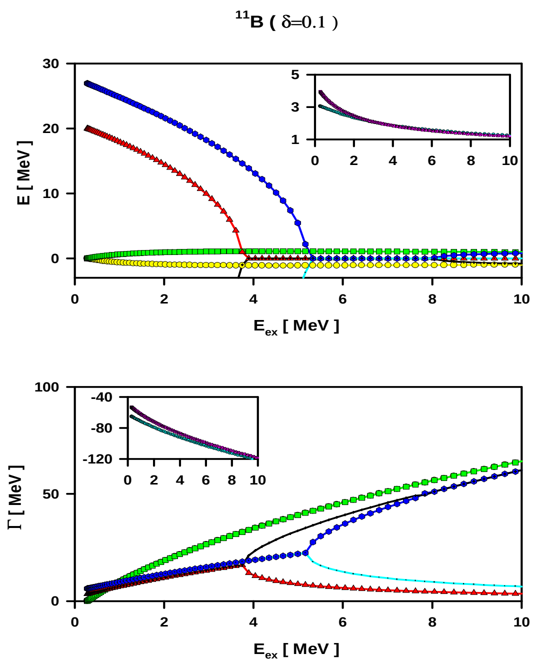

FIG. 1. The centroid energy (above) and the damping width (below) of collective modes vs. excitation energy $E_{\rm ex} = \frac{1}{4}(E/A)$ (above Fermi energy) found for ^{11}B situation. The isovector mode (rhombus) vanishes at about 5 MeV while the corresponding isoscalar mode (triangles) already disappears at 4 MeV. Connected with the disappearing the corresponding damping becomes twofolded. This is explained by the symmetric negative modes plotted as thin lines which disappear as well. A soft mode (squares) arises and is connected with an continuous increasing damping width. The inset gives the instable modes separately.

Besides the standard isovector and isoscalar modes we observe a build up of a very soft mode with a centroid energy around 1 MeV. This mode appears due to the collisional coupling $\bar{c}_{ab}^2$ of (16). When we turn off the relaxation times, i.e. the collision integral, this mode is vanishing as well as in symmetric nuclear matter, see discussion after (16). It shows that this mode appears due to collisional coupling of isovector and isoscalar modes. The corresponding damping of the crossed mode is con-

tinuously increasing with temperature.

Due to the consideration of memory (frequency dependent) relaxation times we observe further an unstable mode shown in the inset of figure 1. If we neglect this memory effect we would not observe this instable modes but the soft mode described above remains.

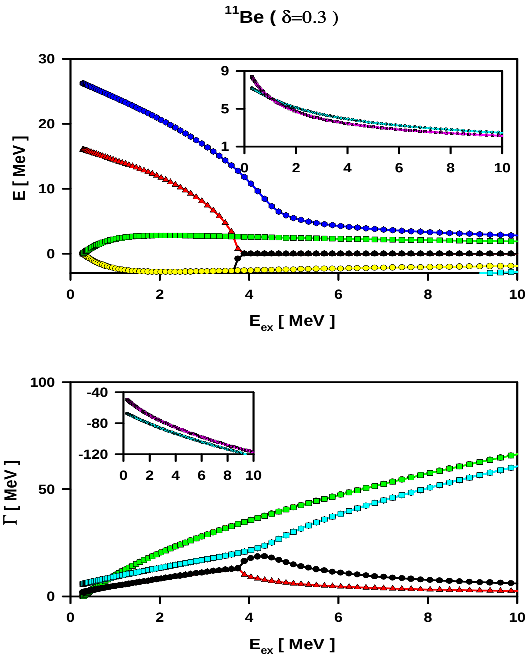

FIG. 2. The centroid energy (above) and the damping width (below) of collective modes vs. excitation energy (above Fermi energy) found for ^{11}Be situation with the same set up as figure 1. The isovector mode (rhombus) decreases and remain constant at about 5 MeV while the corresponding isoscalar mode (triangles) already disappears at 4 MeV. A soft mode (squares) appears with higher centroid energy than in ^{11}B of figure 1 while the damping width is the same.

Looking at a more asymmetric situation of ^{11}Be in figure 2 we see that the centroid energy of the third mode becomes larger while the damping remains the same. A new feature appears for the isovector mode. While in weak asymmetric systems the mode disappear sharply at a certain temperature, we see now a decrease and a convergence towards the new soft mode. This suggest that the new soft mode should be of isovector character. This behavior of the isovector mode is connected with no transition to a twofolded damping we have seen for the more symmetric case. Instead the damping of isovector modes show a transition from a T^2 temperature behavior typical for zero sound damping towards a $E^{1/4}$ behavior typical for first sound damping. We like to pronounce that in symmetric nuclei we did not observe this transition because the mode itself, i.e. the centroid energy is vanishing at the point where this transition behavior of the damping occurs. In asymmetric nuclear matter it should be possible to observe this transition from zero to first sound because the energy of the isovector mode remains finite. The twofold damping of isoscalar mode is modified with respect to the symmetric case but not removed.

One may argue whether this third mode can really appear in the system. A simple consideration may convince us about the possible existence of such mode. Let us assume a coupled set of two type of harmonic oscillators (neutrons and protons) interacting between the same sort of particles with strength k_n and k_p, respectively and between different sorts with k_{np}. Let us choose for simplicity only two neutrons and two protons. Then we obtain the coupled system of harmonic oscillators with frequencies $\omega_n^2 = k_n/m$, $\omega_p^2 = k_p/m$ and $\omega_{np}^2 = k_{np}/m$. The solution yields three basic modes in the system, i.e. $\omega^2 = 2(\omega_n^2 + \omega_{np}^2), 4\omega_{np}^2, 2(\omega_p^2 + \omega_{np}^2)$. If we neglect the different coupling between neutrons and protons ω_{np} we only obtain two modes analogously to isovector and isoscalar ones. We see that the coupling between neutrons and protons can lead to the appearance of a third mode.

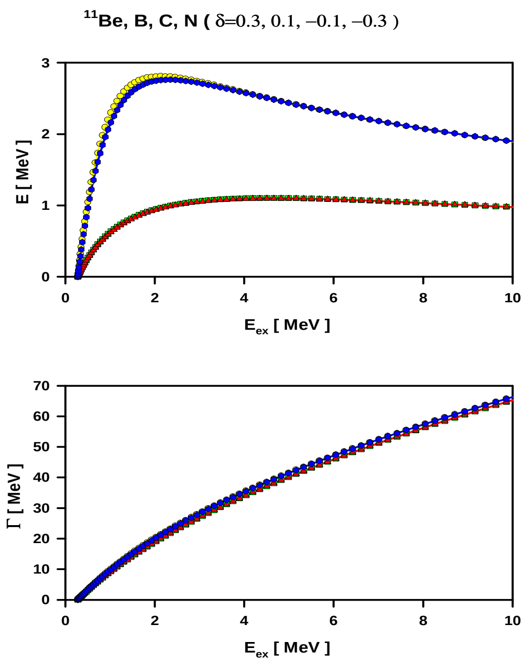

FIG. 3. The centroid energy (above) and the damping width (below) of collective modes vs. excitation energy (above Fermi energy) for the new soft mode for different isobaric states of ^{11}Be. The symmetric nuclei ^{11}Be (circles) and ^{11}N (rhombus) are almost equal and three times larger in the centroid energy than the more symmetric cases ^{11}B (triangles) and ^{11}C (squares).

We conclude that due to collisional coupling there

should be a third soft collective mode observable besides the isovector and isoscalar modes. To study further these modes may open an exciting access to correlation effects in nuclear matter. In figure 3 we give the predictions for the isobar states of ^{11}Be. We find that for ^{11}B and ^{11}C the modes are by a factor three lower. Interesting to remark that the damping width is found to be independent of the asymmetry.

Let us now compare the found new mode with the experimental evidence. There are some hints for a soft mode in ^{11}Be [12]. The authors have observed a low lying structure at around 6 MeV excitation energy with a damping of around 1 MeV which has not been reproduced yet even within refined coupled channel calculations [13]. A standard explanation would give as the origin a weekly bounded single particle neutron orbital. The observed broad structure at 6 MeV might be explained possibly as the here presented coupled mode. The centroid energy as well as damping width at least seem to suggest this interpretation.

Again, we like to point out that the presented results are limited to a pure liquid drop model. We have neglected shell effects and surface effects which are important for realistic calculations in small systems like ^{11}Be.

To summarize we have observed that due to correlational coupling there can exist a new mode which appears besides isovector and isoscalar modes in asymmetric nuclear matter due to collisions. We suggest that this mode may be possible to observe as a soft collective excitation in asymmetric systems. The transition from zero sound damping to first sound damping behavior should become possible to observe for isovector modes since they do not vanish at this transition temperature like in symmetric matter.

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